

BCOM 4TH SEMESTER
QUANTITATIVE TECHNIQUES FOR BUSINESS

UNIT V

CHI-SQUARE TEST (χ^2 -Test)

- The value of chi-square describes the magnitude of difference between observed frequencies and expected frequencies under certain assumptions.
- χ^2 value (χ^2 quantity) ranges from zero to infinity. It is zero when the expected frequencies and observed frequencies completely coincide. So greater the value of χ^2 , greater is the discrepancy between observed and expected frequencies
- χ^2 test is a statistical test which tests the significance of difference between observed frequencies and corresponding theoretical frequencies of a distribution without any assumption about the distribution of the population.
- This is one of the simplest and most widely used nonparametric test in statistical work.
- This test was developed by Prof. Karl Pearson in 1900.
Uses of χ^2 - test

Uses of chi-square test

1. Test of goodness of fit:- χ^2 - test can be used to test whether there is goodness of fit between the observed frequencies and expected frequencies.
2. Test of independence of attributes:- χ^2 test can be used to test whether two attributes are associated or not.
3. Test of homogeneity:- χ^2 -test is very useful to test whether two attributes are homogeneous or not.
4. Testing given population variance:- χ^2 test can be used for testing whether the given population variance is acceptable on the basis of samples drawn from that population.

χ^2 -test as a test of goodness of fit:

As a non-parametric test, χ^2 -test is mainly used to test the goodness of fit between the observed frequencies and expected frequencies.

Procedure

1. Set up null hypothesis that there is goodness of fit between observed and expected frequencies. 2. Find the χ^2 value using the following formula:-
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
 ,Where O = Observed frequencies E = Expected frequencies
2. Compute the degree of freedom. d. f. = n – r – 1 Where 'r' is the number of

- independent constraints to be satisfied by the frequencies
- Obtain the table value corresponding to the level of significance and degrees of freedom.
 - Decide whether to accept or reject the null hypothesis. If the calculated value is less than the table value, we accept the null hypothesis and conclude that there is goodness of fit. If the calculated value is more than the table value we reject the null hypothesis and conclude that there is no goodness of fit.

Do by YOURSELF

Qn No:1 A sample analysis of examination result of 200 students were made. It was found that 46 students had failed, 68 secured IIIrd class, 62 IInd class and the rest were placed in the Ist class. Are these figures commensurate with the general examination results which is in the ratio of 2 : 3: 3: 2 for various categories respectively?

Qn No:2 Test whether the accidents occur uniformly over week days on the basis of the following information:

Days of the week:	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents:	11	13	14	13	15	14	18

χ^2 – test as a test of independence:

χ^2 – test is used to find out whether one or more attributes are associated or not.

Procedure

- Set up null and alternative hypothesis.
 H_0 : Two attributes are independent (i.e., there is no association between the attributes)
 H_1 : Two attributes are dependent (i.e., there is an association between the attributes)
- Find the χ^2 value.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$
- Find the degree of freedom d.f. = (r-1)(c-1)
 Where r = Number of rows c = Number of columns
- Obtain table value corresponding to the level of significance and degree of freedom.
- Describe whether to accept or reject the H_0 . If the calculated value is less than the table value, we accept the H_0 and conclude that the attributes are independent. If the H_0 and conclude that the attributes are dependent.

Do by YOURSELF

Qn No:1 The following table gives data regarding election to an office:-

Attitude towards election

Economic Status

	Rich	Poor	Total
Favourable	50	155	205
Non favourable	90	110	200
Total	140	265	405

Qn No:2 Is attitude towards election influenced by economic status of workers?

In a sample study about the tea habit in two towns, following data are observed in a sample of size 100 each.

Town A:- 51 persons were male, 31 were tea drinkers and 19 were male tea drinks.

Town B:- 46 persons were male, 17 were male tea drinkers and 26 were tea drinkers. Is there any association between sex and tea habits? If so, in which town it is greater?

χ^2 – test as a test of homogeneity

χ^2 – test is used to find whether the samples are homogeneous as far as a particular attribute is concerned.

Procedure

1. Set up null and alternative hypotheses:
 H₀: There is homogeneity.
 H₁: There is no homogeneity (heterogeneity)
2. Find the χ^2 value using the same formula
3. Find the degree of freedom d.f. = (r-1)(c-1)
4. Obtain the table value
5. Decide whether to accept or reject the null hypothesis.

Do by YOURSELF

QN No:1 From the adult population of four large cities, random samples were

selected and the number of married and unmarried men were recorded:

Married	A	B	C	D	Total
	137	164	152	147	600
Single	32	57	56	35	180
Total	169	221	208	182	780

Is there significant variation among the cities in the tendency of men to marry?

Limitations of Chi-square tests:-

1. It is not as reliable as a parametric test. Hence it should be used only when parametric tests cannot be used.
2. χ^2 value cannot be computed when the given values are proportions or percentages.

ANALYSIS OF VARIANCE

Analysis of variance may be defined as a technique which analyses the variance of two or more comparable series (or samples) for determining the significance of differences in their arithmetic means and for determining whether different samples under study are drawn from same population or not, with the of the statistical technique, called F – test.

Characteristics of Analysis of Variance:

1. It makes statistical analysis of variance of two or more samples.
2. It tests whether the difference in the means of different sample is due to chance or due to any significance cause.
3. It uses the statistical test called, F – Ratio.

Types of Variance Analysis:

There are two types of variance analysis. They are:-

1. One way Analysis of Variance
2. Two way analysis of Variance

ONE WAY ANALYSIS OF VARIANCE

- In one way analysis of variance, observations are classified into groups on the basis of a single criterion.
- For example, yield of a crop is influenced by quality of soil, availability of rainfall, quantity of seed, use of fertilizer, etc.
- If we study the influence of one factor, it is called one way analysis of variance.
- If we want to study the effect of fertilizer of yield of crop, we apply different kinds of fertilizers on different paddy fields and try to find out the difference in the effect of these different kinds of fertilizers on yield.

Procedure

1. Set up null and alternative hypothesis:

H_0 : There is no significant difference.

H_1 : There is significant difference.

2. Compute sum of squares Total (SST) $SST = \text{Sum of squares of all observations} - T^2/N$.
3. Compute sum of squares between samples (SSC)

$$SSC = \frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_2} + \frac{(\sum X_3)^2}{n_3} \dots \dots \dots - T^2/N$$

4. Compute sum of squares within sample (SSE)

$$SSE = SST - SSC$$
5. Compute MSC

$$MSC = \frac{SSC}{d.f} = \frac{SSC}{c-1}$$
6. Compute MSE

$$MSE = \frac{SSE}{d.f} = \frac{SSE}{N-c}$$
7. Compute F – ratio:

$$F = \frac{\text{Larger Variance}}{\text{Smaller Variance}}$$
8. Incorporate all these in an ANOVA TABLE as follows:

ANOVA TABLE				
Source of variation	Sum of squares	Degree of freedom	Mean square	F-Ratio
Between Samples	SSC	C-1	$MSC = \frac{SSC}{c-1}$ $MSE = \frac{SSE}{N-c}$	$F = \frac{\text{Larger Variance}}{\text{Smaller Variance}}$
Within Sample	SSE	N-C		
Total	SST	N-1		

9. Obtain table value at corresponding to the level of significance and for degree of freedom of (C-1, N-C).
10. Decide whether to accept or reject the null hypothesis.

Do by YOURSELF

Qn No:1 Given below are the yield (in Kg.) per acre for 5 trial plots of 4 varieties of treatments.

Plot Name	Treatment			
	1	2	3	4
A	42	48	68	80
B	50	66	52	94
C	62	68	76	78
D	34	78	64	82
E	52	70	70	66

Carry out an analysis of variance and state whether there is any significant difference in treatments.

TWO WAY ANALYSIS OF VARIANCE

Two way analysis of variance is used to test the effect of two factors simultaneously on a particular variable.

Procedure:-

1. Set up null and alternative hypothesis.
 H_0 : There is no significant difference between columns. There is no significant difference between rows.
 H_1 : There is significant difference between columns. There is significant difference between rows.
2. Compute SST
 $SST = \text{Sum of squares of all observations} - T^2/N$
3. Compute SSC
 $SSC = \frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_2} + \frac{(\sum X_3)^2}{n_3} \dots \dots \dots - T^2/N$
4. Compute SSR
 $SSR = \frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_2} + \frac{(\sum X_3)^2}{n_3} \dots \dots \dots - T^2/N$
 Here $\sum X_1, \sum X_2$, etc denote the row totals
5. Compute SSE
 $SSE = SST - (SSC + SSR)$
6. Compute MSC
 $MSC = \frac{SSC}{d.f} = \frac{SSC}{c-1}$
7. Compute MSR
 $MSR = \frac{SSR}{d.f} = \frac{SSR}{r-1}$
8. Compute MSE
 $MSE = \frac{SSE}{d.f} = \frac{SSE}{(c-1)(r-1)}$
9. Compute F – ratio in respect of columns
 $F_c = \frac{MSC}{MSE}$
10. Compute F – ratio in respect of rows
 $F_r = \frac{MSR}{MSE}$
11. Obtain the table value
12. Decide whether to accept or reject the H_0

TWO WAY ANOVA TABLE					
Source of variation	Sum of squares	Degree of freedom	Mean square	F-Ratio	
Between Columns	SSC	c-1	$MSC = \frac{SSC}{c-1}$	$F = \frac{MSC}{MSE}$	
Between Rows	SSE	r-1	$MSR = \frac{SSR}{r-1}$		
Residual		(c-1)(r-1)	$MSE = \frac{SSE}{(c-1)(r-1)}$		

Total	SST	N-1		
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Do by YOURSELF

Qn No:1 Apply the technique of analysis of variance to the following data relating to yields of 4 varieties of wheat in 3 blocks:

Plot Name	Blocks		
	X	Y	Z
A	42	48	68
B	50	66	52
C	62	68	76
D	34	78	64
E	52	70	70

NB: To solve the problems, follow the steps given in the procedure

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