BBA 4th SEMESTER

MANAGEMENT SCIENCE

UNIT II

LINEAR PROGRAMMING PROBLEM

Formulation of LPP

The Diet Problem

- In the diet model, a list of available foods is given together with the nutrient content and the cost per unit weight of each food. A certain amount of each nutrient is required per day.
- **Example**: Here is the data corresponding to a civilization with just two types of grains (G1 and G2) and three types of nutrients (starch, proteins, vitamins): Starch Proteins Vitamins Cost ($/kg) G1 5 4 2 0.6 G2 7 2 1 0.35 Nutrient content and cost per kg of food. The requirement per day of starch, proteins and vitamins is 8, 15 and 3 respectively. The problem is to find how much of each food to consume per day so as to get the required amount per day of each nutrient at minimal cost.
- When trying to formulate a problem as a linear program, the first step is to decide which **decision variables** to use. These variables represent the unknowns in the problem. In the diet problem, a very natural choice of decision variables is x_1: number of units of grain G1 to be consumed per day, x_2: number of units of grain G_2 to be consumed per day.
- The next step is to write down the **objective function**. The objective function is the function to be minimized or maximized. In this case, the objective is to minimize the total cost per day which is given by $Z= 0.6x_1 + 0.35x_2$ (the value of the objective function is often denoted by Z).
- Finally, we need to describe the different **constraints** that need to be satisfied by x_1 and x_2. First of all, x_1 and x_2 must certainly satisfy $x_1 \geq 0$ and $x_2 \geq 0$.
- Not all possible values for x_1 and x_2 give rise to a diet with the required amounts of nutrients per day.
- The amount of starch in x1 units of G1 and x2 units of G2 is $5x_1 + 7x_2$ and this amount must be at least 8, the daily requirement of starch. Therefore, x_1 and x_2 must satisfy $5x_1 + 7x_2 \geq 8$.
- Similarly, the requirements on the amount of proteins and vitamins imply the constraints $4x_1 + 2x_2 \geq 15$ and $2x_1 + x_2 \geq 3$.
- This diet problem can therefore be formulated by the following linear program:
  Minimize $z = 0.6x_1 + 0.35x_2$
  subject to:
  $5x_1 + 7x_2 \geq 8$
  $4x_1 + 2x_2 \geq 15$
  $2x_1 + x_2 \geq 3$

$x_1 \geq 0, x_2 \geq 0.$

- A solution $x = (x_1, x_2)$ is said to be feasible with respect to the above linear program if it satisfies all the above constraints.
- The set of feasible solutions is called the feasible space or feasible region. A feasible solution is optimal if its objective function value is equal to the smallest value z can take over the feasible region.

Do it YOURSELF

Qn No:1 A paint manufacturer produces two types of paint, one type of standard quality (S) and the other of top quality (T). To make these paints, he needs two ingredients, the pigment and the resin. Standard quality paint requires 2 units of pigment and 3 units of resin for each unit made, and is sold at a profit of R1 per unit. Top quality paint requires 4 units of pigment and 2 units of resin for each unit made, and is sold at a profit of R1.50 per unit. He has stocks of 12 units of pigment, and 10 units of resin. Formulate the above problem as a linear programming problem to maximize his profit?

Qn No:2 A house wife wishes to mix two types of food F1 and F2 in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food F1 costs E60/Kg and Food F2 costs E80/kg. Food F1 contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while Food F2 contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Formulate this problem as a linear programming problem to minimize the cost of the mixtures.

Graphical Solution to LPP

The graphical method of solving a linear programming problem is used when there are only two decision variables. If the problem has three or more variables, the graphical method is not suitable. In that case we use the simplex method

1. **Solution** A set of values of decision variables satisfying all the constraints of a linear programming problem is called a solution to that problem.
2. **Feasible solution** Any solution which also satisfies the non-negativity restrictions of the problem is called a feasible solution.
3. **Optimal feasible solution** Any feasible solution which maximizes or minimizes the objective function is called an optimal feasible solution.
4. **Feasible region** The common region determined by all the constraints and non-negativity restriction of a LPP is called a feasible region.
5. **Corner point** A corner point of a feasible region is a point in the feasible region that is the intersection of two boundary lines.

If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one (or more) of the corner points of the feasible region.

Graphical method of solving a LPP

(1) Formulate the linear programming problem.

(2) Graph the feasible region and find the corner points. The coordinates of the corner

points can be obtained by either inspection or by solving the two equations of the lines intersecting at that point.

(3) Make a table listing the value of the objective function at each corner point.

(4) Determine the optimal solution from the table in step 3. If the problem is of maximization (minimization) type, the solution corresponding to the largest (smallest) value of objective function is the optimal solution of the LPP.

Qn No: 1

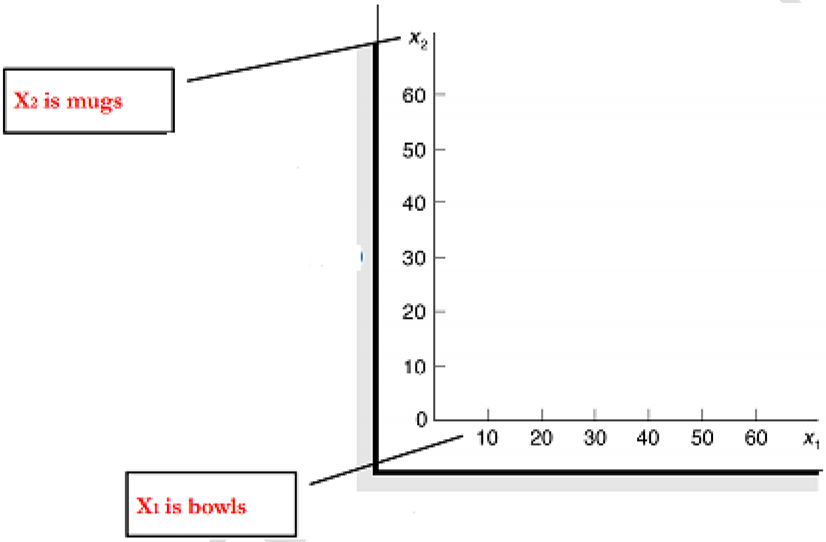
Maximize $Z = 40x_1 + 50x_2$

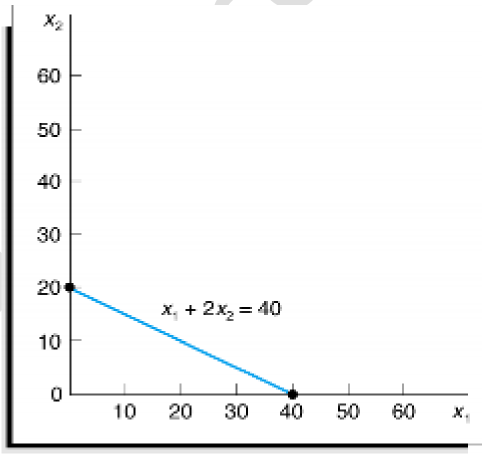
subject to:

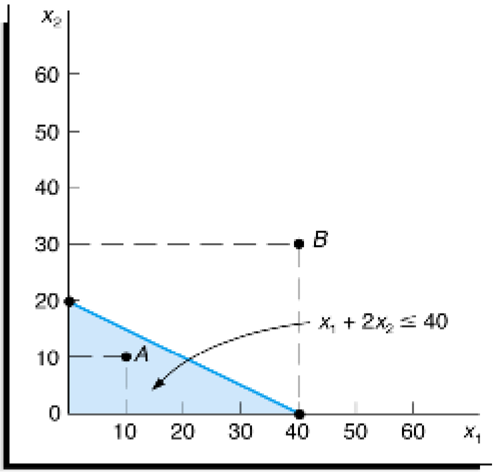
$1x_1 + 2x_2 \leq 40$

$4x_1 + 3x_2 \leq 120$

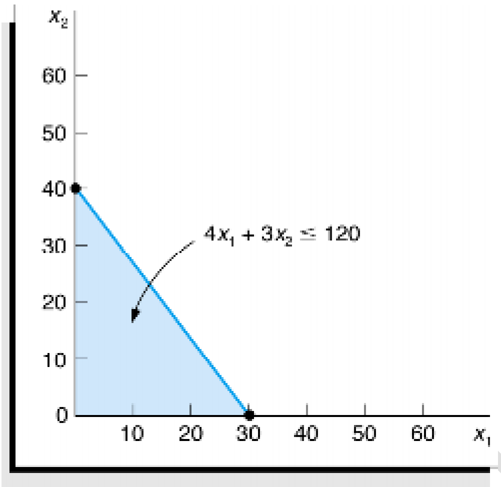
$x_1, x_2 \geq 0$



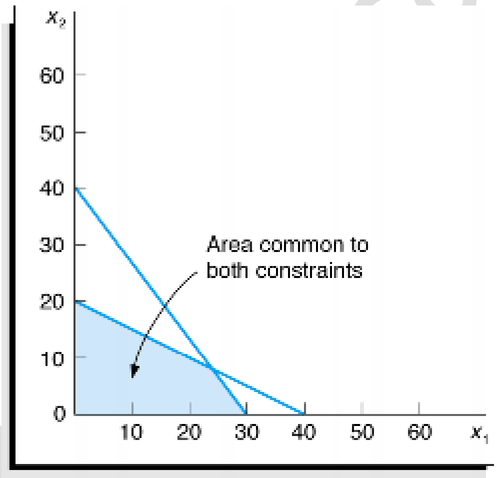




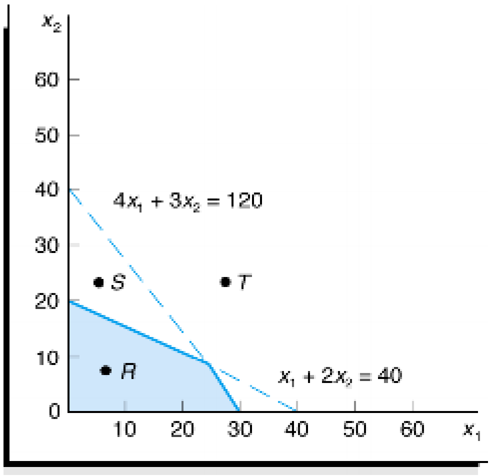
x₂
60
50
40
30
20
10
0
10
20
30
40
50
60
x₁
B
A
$x_1 + 2x_2 \leq 40$

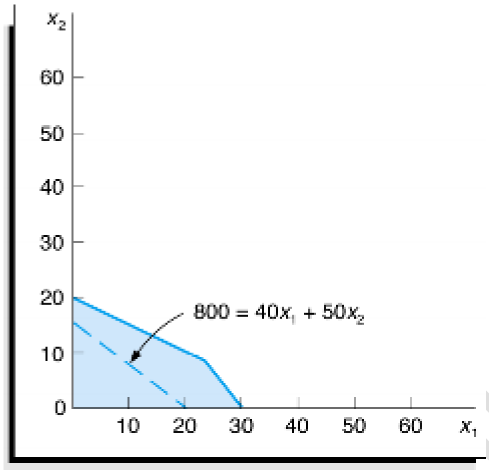


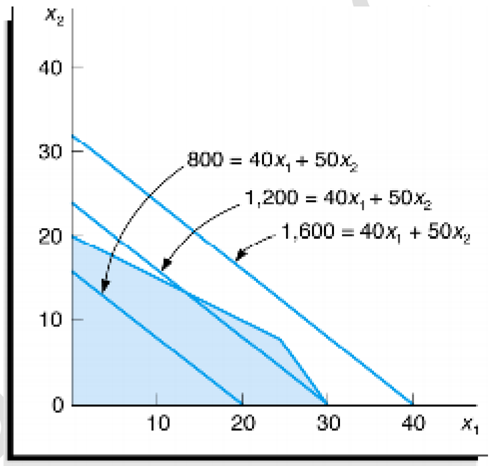
x₂
60
50
40
30
20
10
0
10
20
30
40
50
60
x₁
$4x_1 + 3x_2 \leq 120$

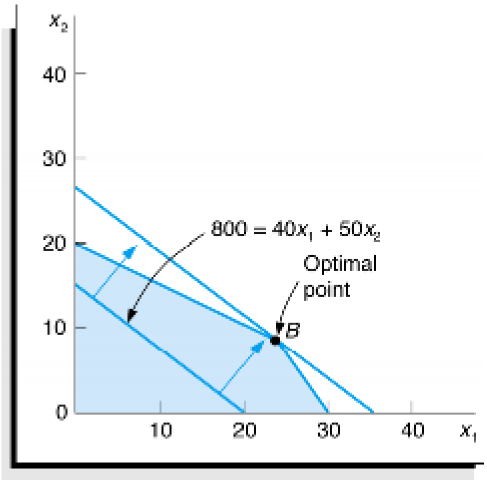


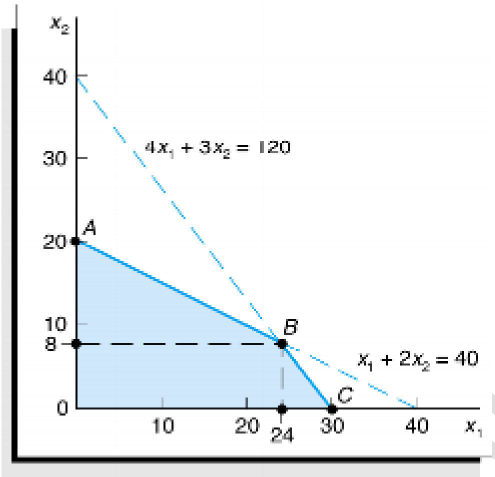
x₂
60
50
40
30
20
10
0
10
20
30
40
50
60
x₁
Area common to
both constraints

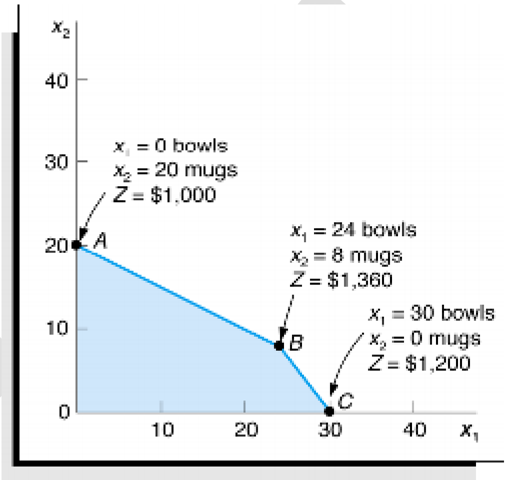

x_2
60
50
40
30
20
10
0
$4x_1 + 3x_2 = 120$
S
T
R
$x_1 + 2x_2 = 40$
10
20
30
40
50
60
x_1

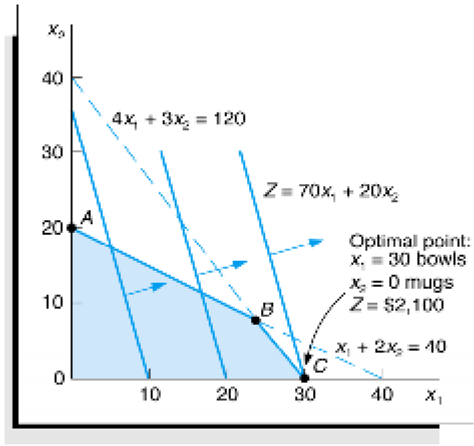

x_2
60
50
40
30
20
10
0
$800 = 40x_1 + 50x_2$
10
20
30
40
50
60
x_1


x_2
40
30
20
10
0
$800 = 40x_1 + 50x_2$
$1,200 = 40x_1 + 50x_2$
$1,600 = 40x_1 + 50x_2$
10
20
30
40
x_1


x_2
40
30
20
10
0
10
20
30
40
x_1
$800 = 40x_1 + 50x_2$
Optimal point
B


x_2
40
30
20
10
8
0
10
20
24
30
40
x_1
$4x_1 + 3x_2 = 120$
$x_1 + 2x_2 = 40$
A
B
C


x_2
40
30
20
10
0
10
20
30
40
x_1
x_1 = 0 bowls
x_2 = 20 mugs
Z = $1,000
A
x_1 = 24 bowls
x_2 = 8 mugs
Z = $1,360
B
x_1 = 30 bowls
x_2 = 0 mugs
Z = $1,200
C



Qn No:2 Solve the following problem graphically.

Maximize Z = 60x1 + 40x2

Subject to:

2x1 + x2 ≤ 60

x1 ≤ 25

x2 ≤ 35

x1 ≥ 0, x2 ≥ 0

Solution

- 2x1 + x2 = 60
  x1 = 25
  x2 = 35
  x1 = 0, x2 = 0 1.
- 2x1 + x2 = 60
  Let x1 = 0
  then 2 × 0 + x2 = 60
  x2 = 60 **(0, 60)**
  Let x2 = 0
  then 2x1 + 0 = 60
  2x1 = 60
  x1 = . 60/2 = 30 **(30, 0)**
  P = (0, 35); S = (25, 0)

Points	x1	x2	Z = 60x1 + 40x2
0	0	0	0
P	O	35	60 × 0 + 40 × 35 = 1400
Q	12.5	35	60 × 12.5 + 40 × 35 = 2150
R	25	10	60 × 25 + 40 × 10 = 1900
S	25	0	60 ×25 + 40 × 10 = 1500

Solution is Q = (12.5, 35) and Z = 2150

Point Q passes through two straight lines.

x2 = 35

2x1 + x2 = 60

2x1 + 35 = 60

2x1 = 60 - 35 = 25

x1 = 25/2 = 12.5

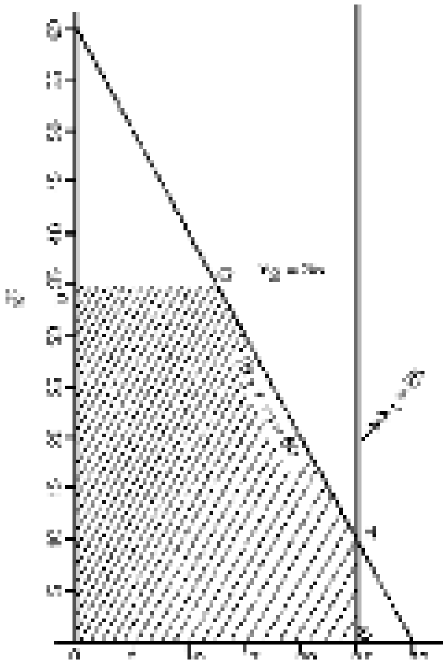
Point R passes through two straight lines.

x1 = 25

2x1 + x2 = 60

2 × 25 + x2 = 60

x2 = 60- 50 = 10



Do by YOURSELF

Minimize $Z = 20x_1 + 40x_2$

Subject to:

$36x_1 + 6x_2 \geq 108$

$3x_1 + 12x_2 \geq 36$

$20x_1 + 10x_2 \geq 100$

$x_1, x_2 \geq 0$